1. Consider a radioactive nuclide of element X with mass number A and atomic number Z. Write a general nuclear equation for each type of decay in the table below.

Decay Type	General Nuclear Equation	Description
β decay	${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}\beta$	Neutron-rich
positron emission	${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}\beta$	Neutron-poor
electron capture	${}_{-1}^{0}\beta + {}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y$	Neutron-poor
α decay	${}^{A}_{Z}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}\alpha$	Neutron-rich (Z>83)

2. Complete each nuclear reaction given below.

A) ${}^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + \boxed{{}^{4}_{2}\alpha}$

B) ${}^{131}_{53}I \rightarrow {}^{131}_{54}Xe + \boxed{{}_{-1}^{0}\beta}$

C) ${}^{11}_{6}C \rightarrow \boxed{{}^{11}_{5}B} + {}_{+1}^{0}\beta$

D) Electron capture by cadmium-104 (${}^{104}_{48}Cd$) ${}^{104}_{48}Cd + {}_{-1}^{0}\beta \rightarrow {}^{104}_{47}Ag$

E) ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow \boxed{{}^{236}_{92}U} \rightarrow {}^{137}_{52}Te + \boxed{{}^{97}_{40}Zr} + 2{}^{1}_{0}n$

3. Both carbon-14 and potassium-40 can be used for radiometric dating. The half-life of ^{14}C is 5730 years and the half-life of ^{40}K is 1.28×10^9 years.

$$\text{Rate} = kN \qquad t_{1/2} = \frac{\ln 2}{k} \qquad t = -\frac{1}{k}\ln\frac{N_t}{N_0}$$

A) Which radioisotope is preferred for radiodating a rock that is 20,000 years old?

^{14}C because more half-lives have past and so there is a more appreciable/measurable ratio.

B) Neither method is good for a 200,000-year-old rock. Calculate the fraction of ^{14}C and ^{40}K remaining in the rock to determine why this is so.

$$\frac{N_t}{N_0} = 0.5^{t/t_{1/2}} \quad \rightarrow \quad {}^{14}C\!: \frac{N_t}{N_0} = 0.5^{\frac{200,000}{5730}} = 3.1 \times 10^{-11} \qquad {}^{40}K\!: \frac{N_t}{N_0} = 0.5^{\frac{200,000}{1.28\times10^{9}}} = 1.00$$

Too many half-lives have passed for ^{14}C and too few for ^{40}K for any measurable ratio.

4. Mercury-197 has a half-life of 65 hours. What fraction of a mercury sample remains after 6 days?

$$t = -\frac{1}{k}\ln\frac{N_t}{N_0}$$

$$6 \text{ days} \times \frac{24 \text{ hr}}{1 \text{ day}} = -\frac{65 \text{ hr}}{\ln 2}\ln\frac{N_t}{N_0}$$

$$\ln\frac{N_t}{N_0} = -1.53_6$$

$$\frac{N_t}{N_0} = 0.22 \ (22\ \%)$$

5. Iron-56 is often considered the most stable nuclide although it is actually the third-most stable. Nickel-62 is the most stable nuclide. Given the mass of a proton, neutron, and measured mass of $^{62}_{28}\text{Ni}$ below, calculate the binding energy *per* nucleon for $^{62}_{28}\text{Ni}$.

$m_{\text{proton}} = 1.0073$ amu $\quad m_{\text{neutron}} = 1.0087$ amu $\quad m_{^{62}_{28}\text{Ni}} = 61.9283$ amu

Recall $\Delta E = \Delta mc^2$ where $c = 3.00 \times 10^8$ m/s, 1 amu $= 1.66 \times 10^{-27}$ kg, and $1\text{ J} = 1\text{ kg} \cdot (\text{m}/\text{s})^2$.

$^{62}_{28}\text{Ni}$ has 28 protons and 34 neutrons, a total of 62 nucleons. So, the mass defect is:

$$\Delta m = (\text{measured mass}) - (\text{expected mass})$$
$$= m_{^{62}_{28}\text{Ni}} - [28 \times m_{\text{protons}} + 34 \times m_{\text{neutrons}}]$$
$$= 61.9283 \text{ amu} - [28 \times (1.0073 \text{ amu}) + 34 \times (1.0087 \text{ amu})]$$
$$\Delta m = -0.5719 \text{ amu}$$

Therefore, the binding energy per nickel-62 nucleus is:

$$\text{BE} = |\Delta m| c^2$$
$$= \left| -0.5719 \text{ amu} \times \frac{1.66 \times 10^{-27} \text{kg}}{1 \text{ amu}} \right| \times \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2$$
$$\text{BE} = 8.544 \times 10^{-11} \text{ J}$$

And, the binding energy per nucleon is 1.378×10^{-12} J/nucleon.

6. Silicon-28 can be made by many different nuclear fusion reactions. Which of the two fusion reactions, A or B, releases the greater amount of energy?

A) $\boxed{^{14}\text{N} + {}^{14}\text{N} \rightarrow {}^{28}\text{Si}}$ where: $^{14}\text{N} = 14.00307$ amu $\quad ^{28}\text{Si} = 27.97693$ amu

B) $^{16}\text{O} + {}^{12}\text{C} \rightarrow {}^{28}\text{Si}$ $\quad ^{16}\text{O} = 15.99491$ amu $\quad ^{12}\text{C} = 12.00000$ amu

The magnitude of the mass defect ($|\Delta m|$) is proportional to the energy released.

A) $\Delta m = m_{\text{Si}} - 2 \times m_{\text{N}} = 27.97693 \text{ amu} - 2 \times (14.00307 \text{ amu}) = -0.02921 \text{ amu}$

B) $\Delta m = m_{\text{Si}} - [m_{\text{O}} + m_{\text{C}}] = 27.97693 \text{ amu} - [15.99491 + 12.00000] \text{ amu} = -0.01798 \text{ amu}$

Computing the energies released directly is also okay:

$$\text{A)} \quad \Delta E = \left(-0.02921 \text{ amu} \times \frac{1.66 \times 10^{-27} \text{kg}}{1 \text{ amu}}\right) \times \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 = -4.364 \times 10^{-12} \text{ J}$$

$$\text{B)} \quad \Delta E = \left(-0.01798 \text{ amu} \times \frac{1.66 \times 10^{-27} \text{kg}}{1 \text{ amu}}\right) \times \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 = -2.686 \times 10^{-12} \text{ J}$$

Propose an alternative fusion reaction to produce ^{28}Si.

$$^{24}_{12}\text{Mg} + {}^{4}_{2}\text{He} \rightarrow {}^{28}_{14}\text{Si}$$