SECTIONS D01 & D07
Week 6

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YALE UNIVERSITY

CHEMISTRY 161

FALL 2018

www.mioy.org/chem161

COLLEGE: YOU WILL BE YOUR BEST TEACHER
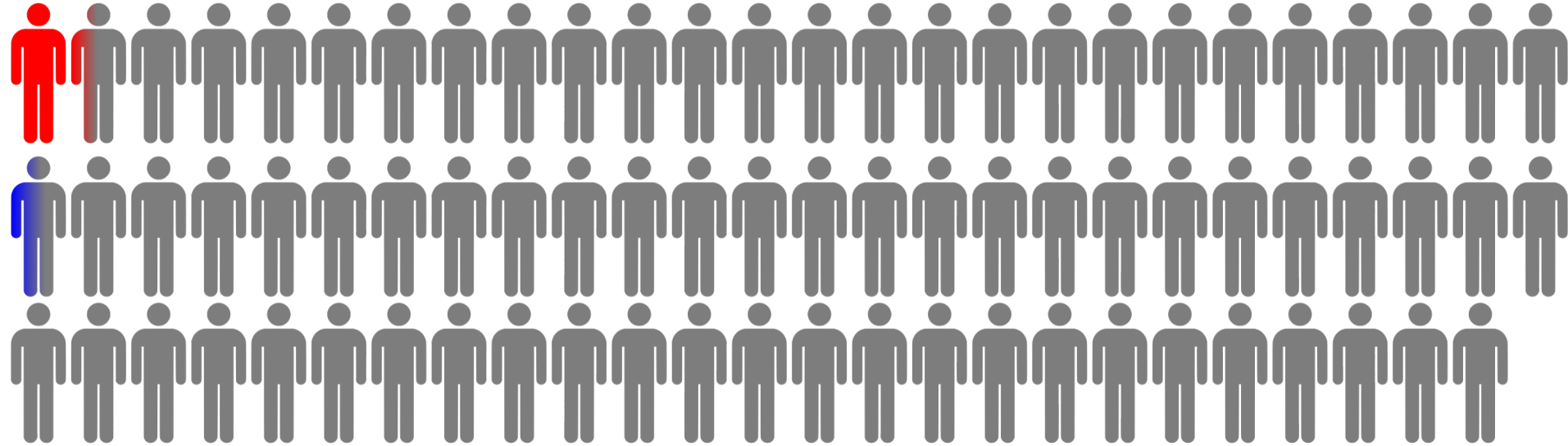
LECTURES
PROF. HOLLAND

1.32 days

DISCUSSIONS
TF

0.45 days

YOU

75.23 days

FALL SEMESTER
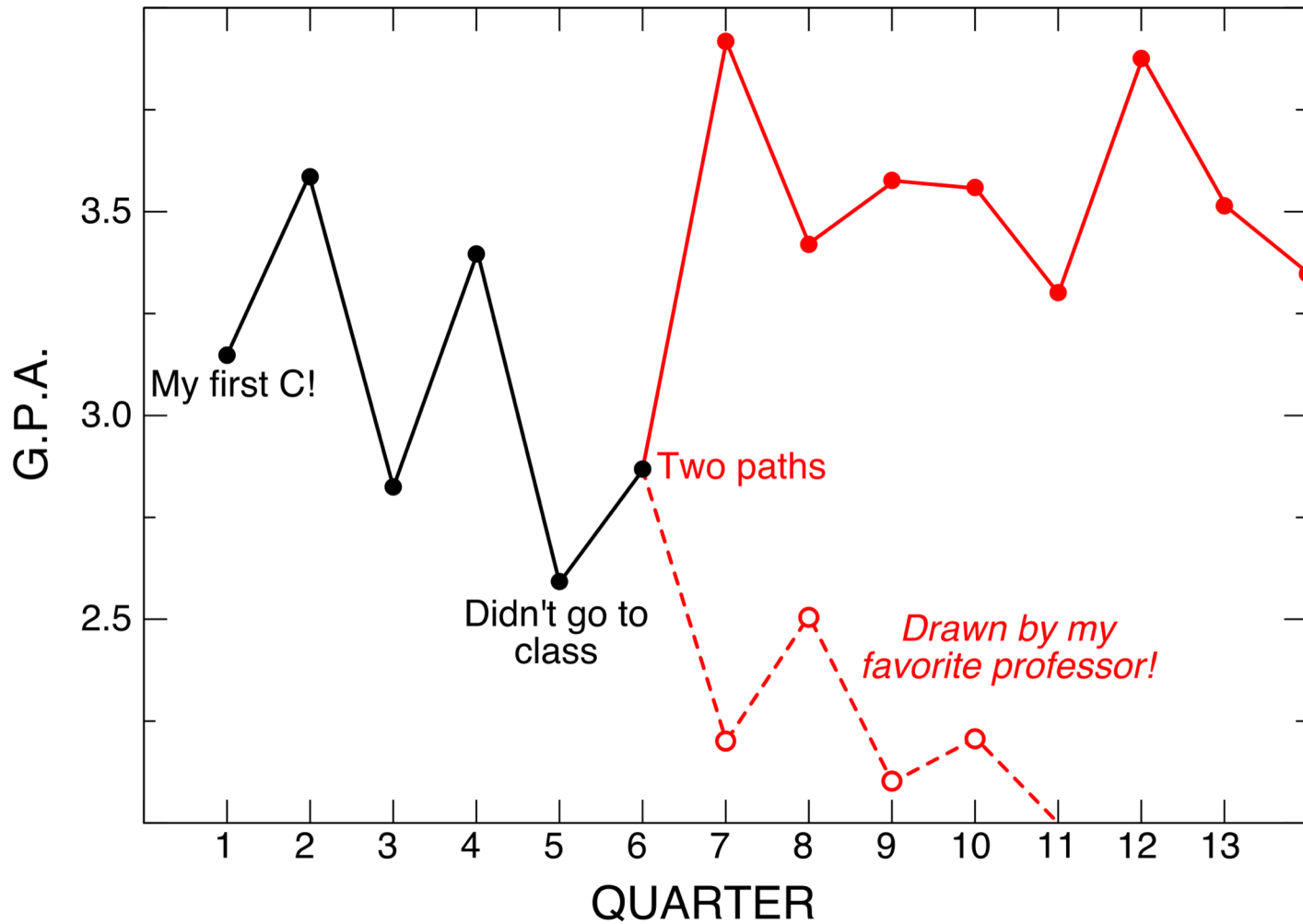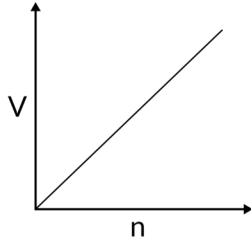77 DAYS TOTAL

~17 days/course

Excludes weekends

Volume vs. Moles (V vs. n)	Pressure vs. Moles (P vs. n)	Pressure vs. Volume (P vs. V)
$V \propto n$ (constant T, P) $$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$	$P \propto n$ (constant T, V) $$\frac{P_1}{n_1} = \frac{P_2}{n_2}$$	$P \propto \dfrac{1}{V}$ (constant n, T) $$P_1 V_1 = P_2 V_2$$
Volume vs. Temp (V vs. T)	Pressure vs. Temp (P vs. T)	P vs. V. vs. T
$V \propto T$ (constant n, P) $$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$	$P \propto T$ (constant n, V) $$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$	$PV \propto T$ (constant n) $$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

THE IDEAL GAS LAW

Gases behave ideally at:
- High temperatures
- Low pressures
- Large volume container (relative to gas particle volume)

UNIVERSAL GAS CONSTANT (R):

From the pairwise gas relationships, we can write:

$$PV \propto nT$$

$$-and-$$

$$\frac{PV}{nT} = R = 0.08206 \; \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}$$

$$\boxed{PV = nRT}$$

Reference Points for Gases

- Standard Temperature and Pressure (STP): P = 1 atm and T = 273 K (0.0 °C)

Molar Volume:
- Volume occupied by one mole of an ideal gas at STP
- V = 22.4 L (calculated from ideal gas law)
- V = (1 atm)/(1 mol)(R)(273 K)

ALWAYS WORK IN ABSOLUTE TEMPERATURE (K SCALE) AND ABSOLUTE PRESSURE SCALE FOR IDEAL GAS LAW!

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$$\boxed{PV = nRT}$$

Determine the size of a balloon filled with 4.00 moles of helium gas if the room is at 1.00 atm and 22 °C.

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What we know: P = 1.00 atm   V = ?
n = 4.00 moles   T = 22 °C = 295.15 K

Set up the ideal gas law and solve for V:

$$PV = nRT$$

$$V = \frac{nRT}{P}$$

$$= \frac{(4.00 \text{ mol}) \left(0.08206 \ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(295.15 \text{ K})}{1.00 \text{ atm}}$$

$$V = 96.9 \text{ L}$$

Volume vs. Moles (V vs. n)

$$V \propto n \ (\text{constant } T, P)$$
$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

I		II		Change		Constant
P_1	=	P_2		V/n	=	RT/P
V_1		V_2				
n_1		n_2				
T_1	=	T_2				

$$\frac{V_1}{n_1} = \frac{RT}{P} = \frac{V_2}{n_2}$$

Pressure vs. Moles (P vs. n)

$$P \propto n \ (\text{constant } T, V)$$
$$\frac{P_1}{n_1} = \frac{P_2}{n_2}$$

I		II		Change		Constant
P_1		P_2		P/n	=	RT/V
V_1	=	V_2				
n_1		n_2				
T_1	=	T_2				

$$\frac{P_1}{n_1} = \frac{RT}{V} = \frac{P_2}{n_2}$$
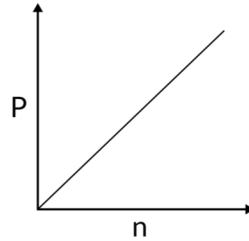
Pressure vs. Volume (P vs. V)

$$P \propto \frac{1}{V} \ (\text{constant } n, T)$$
$$P_1 V_1 = P_2 V_2$$

I		II		Change		Constant
P_1		P_2		PV	=	nRT
V_1		V_2				
n_1	=	n_2				
T_1	=	T_2				

$$P_1 V_1 = nRT = P_2 V_2$$
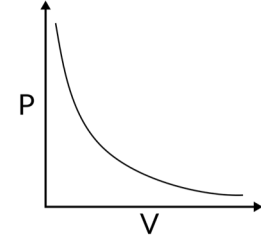
Volume vs. Temp (V vs. T)

$$V \propto T \ (\text{constant } n, P)$$
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

I		II		Change		Constant
P_1	=	P_2		V/T	=	nR/P
V_1		V_2				
n_1	=	n_2				
T_1		T_2				

$$\frac{V_1}{T_1} = \frac{nR}{P} = \frac{V_2}{T_2}$$
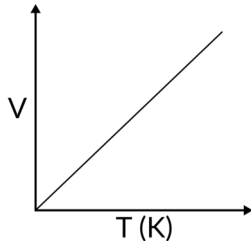
Pressure vs. Temp (P vs. T)

$$P \propto T \ (\text{constant } n, V)$$
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

I		II		Change		Constant
P_1		P_2		P/T	=	nR/V
V_1	=	V_2				
n_1	=	n_2				
T_1		T_2				

$$\frac{P_1}{T_1} = \frac{nR}{V} = \frac{P_2}{T_2}$$
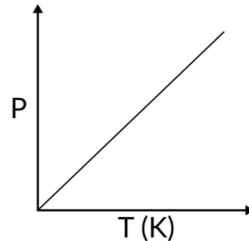
P vs. V. vs. T

$$PV \propto T \ (\text{constant } n)$$
$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

I		II		Change		Constant
P_1		P_2		PV/T	=	nR
V_1		V_2				
n_1	=	n_2				
T_1		T_2				

$$\frac{P_1 V_1}{T_1} = nR = \frac{P_2 V_2}{T_2}$$

Given 6.0 L of nitrogen gas at –25 °C, what volume will the nitrogen gas occupy at 72 °C? *Assume constant pressure.*

Given 6.0 L of nitrogen gas at −25 °C, what volume will the nitrogen gas occupy at 72 °C? *Assume constant pressure.*

First, realize that pressure (P) and number of moles (n) is constant.
If so, accept that volume (V) and temperature (T) change.

What we know: V_1 = 6.0 L V_2 = ?
                              T_1 = −25 °C = 248.$_{15}$ K T_2 = 72 °C = 345.$_{15}$ K

Set up the expression for these conditions and solve for V_2:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{V_1 T_2}{T_1}$$

$$= \frac{(6.0 \text{ L})(345._{15} \text{ K})}{248._{15} \text{ K}}$$

$$V_2 = 8.3 \text{ L}$$

Consider a sample of gas at 2.00 atm in a 35.0 L container at 25 °C. You transfer all of the gas to a 70.0 L container and you heat the gas to 50.0 °C.

Determine the new pressure of the gas.

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Determine the new pressure of the gas.

First, realize that only the number of moles (n) is constant.
If so, accept that pressure (P), volume (V), and temperature (T) all change.

What we know: P_1 = 2.00 atm P_2 = ?
                     V_1 = 35.0 L V_2 = 70.0 L
                     T_1 = 25 °C = 298.$_{15}$ K T_2 = 50.0 °C = 323.1$_5$ K

Set up the expression for these conditions and solve for P_2:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$P_2 = \frac{P_1 V_1 T_2}{T_1 V_2}$$

$$= \frac{(2.00 \text{ atm})(35.0 \text{ L})(323.1_5 \text{ K})}{(298._{15} \text{ K})(70.0 \text{ L})}$$

$$P_2 = \quad 1.08 \text{ L}$$

Consider a sample of gas at 5.40 atm in a 1.00 L <u>rigid</u> container at 40.0 °C. You heat the gas to 80.0 °C.

What happens? Solve for the new condition.

Consider a sample of gas at 5.40 atm in a 1.00 L <u>rigid</u> container at 40.0 °C. You heat the gas to 80.0 °C.
What happens? Solve for the new condition.

First, realize that volume (V) and the number of moles (n) are constant.
If so, accept that pressure (P) and temperature (T) change.

What we know: P_1 = 5.40 atm P_2 = ?
                 T_1 = 40.0 °C = 313.15 K T_2 = 80.0 °C = 353.15 K

Set up the expression for these conditions and solve for P_2:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$P_2 = \frac{P_1 T_2}{T_1}$$

$$= \frac{(5.40 \text{ atm})(353.1_5 \text{ K})}{313.1_5 \text{ K}}$$

$$P_2 = 6.09 \text{ atm}$$

Consider a sample of gas at 5.40 atm in a 1.00 L container fitted with a <u>moveable</u> piston at 40.0 °C. You heat the gas to 80.0 °C.

What happens? Solve for the new condition.

Consider a sample of gas at 5.40 atm in a 1.00 L container fitted with a <u>moveable</u> piston at 40.0 °C. You heat the gas to 80.0 °C.
What happens? Solve for the new condition.

First, realize that pressure (P) and the number of moles (n) are constant.
If so, accept that volume (V) and temperature (T) change.

What we know: V_1 = 1.00 L V_2 = ?
                         T_1 = 40.0 °C = 313.1$_5$ K T_2 = 80.0 °C = 353.1$_5$ K

Set up the expression for these conditions and solve for V_2:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$V_2 = \frac{V_1 T_2}{T_1}$$

$$= \frac{(1.00 \text{ L})(353.1_5 \text{ K})}{313.1_5 \text{ K}}$$

$$\color{red}{V_2 = 1.13 \text{ L}}$$

PARTIAL PRESSURES

- Each gas (A, B, etc) contributes some <u>partial pressure</u> (P_A) to total pressure (P_{total})

$$P_{total} = P_A + P_B + ...$$

How to calculate partial pressure of gas A in a mixture:
- Determine moles of the gas $\qquad\qquad n_A$
- Determine the mole ratio $\qquad\qquad\qquad X_A = n_A/n_{total}$
- Multiply mole ratio by total pressure $\qquad P_A = X_A P_{total}$

A 50.0 L steel tank contains 186 mol N_2 and 145 mol O_2 at 24 °C.
What is the partial pressure of each gas in the tank?
What is the total pressure in the tank?

A 50.0 L steel tank contains 186 mol N_2 and 145 mol O_2 at 24 °C.
What is the partial pressure of each gas in the tank?
What is the total pressure in the tank?

Solve for the partial pressures of each gas using the ideal gas law:
  For N_2:

$$P_{N_2} = \frac{n_{N_2} RT}{V}$$

$$= \frac{(186 \text{ mol}) \left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right) (297._{15} \text{ K})}{50.0 \text{ L}}$$

$$P_{N_2} = 90.7_1 \text{ atm}$$

  For O_2:

$$P_{O_2} = \frac{n_{O_2} RT}{V}$$

$$= \frac{(145 \text{ mol}) \left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right) (297._{15} \text{ K})}{50.0 \text{ L}}$$

$$P_{O_2} = 70.7_1 \text{ atm}$$

For total pressure: $P_{\text{total}} = P_{N_2} + P_{O_2} = 90.7_1 \text{ atm} + 70.7_1 \text{ atm} = 161 \text{ atm}$

A 50.0 L steel tank contains 186 mol N_2 and 145 mol O_2 at 24 °C.
What is the partial pressure of each gas in the tank?
What is the total pressure in the tank?

Alternative Solution: Solve for the partial pressures of each gas using mole fractions.
The total pressure (P_{total}) can be determined from the ideal gas law:

$$P_{total} = \frac{(n_{N_2} + n_{O_2})RT}{V}$$

$$= \frac{(186 \text{ mol} + 145 \text{ mol})\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(297._{15} \text{ K})}{50.0 \text{ L}}$$

$$P_{total} = 161._4 \text{ atm}$$

Find mole fractions for each gas:

$$X_{N_2} = \frac{n_{N_2}}{n_{N_2} + n_{O_2}} = \frac{186 \text{ mol}}{331 \text{ mol}} = 0.561_9 \text{ and } X_{O_2} = \frac{n_{O_2}}{n_{N_2} + n_{O_2}} = \frac{145 \text{ mol}}{331 \text{ mol}} = 0.438_1$$

Find partial pressures from mole fractions and P_{total}:

$$P_{N_2} = X_{N_2}P_{total} = (0.561_9)(161._4 \text{ atm}) = 90.7 \text{ atm} \text{ and } P_{O_2} = X_{O_2}P_{total} = (0.438_1)(161._4 \text{ atm}) = 70.7 \text{ atm}$$

Into an empty, locked (immoveable) piston with a volume of 1.15 L and temperature of 245 K, you inject:

- 589 mL of carbon monoxide gas at 325 K and 1.20 atm
- 473 mL of nitrogen gas at 298 K and 2.60 atm

What is the final pressure of the contents after injection? Assume no reaction occurs.

Into an empty, locked (immoveable) piston with a volume of 1.15 L and temperature of 245 K, you inject:
- 589 mL of carbon monoxide gas at 325 K and 1.20 atm
- 473 mL of nitrogen gas at 298 K and 2.60 atm

What is the final pressure of the contents after injection? Assume no reaction occurs.

Recognize that the final temperature of both gases will be 245 K.
We can use the initial P, V, and T of each gas to find the number of moles of each:

$$n_{CO} = \frac{PV}{RT}$$

$$= \frac{(1.20 \text{ atm}) \left(589 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}}\right)}{\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(325 \text{ K})}$$

$$n_{CO} = 0.0265_0 \text{ mol}$$

$$n_{N_2} = \frac{PV}{RT}$$

$$= \frac{(2.60 \text{ atm}) \left(473 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}}\right)}{\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(2\mathbf{98} \text{ K})}$$

$$n_{N_2} = 0.0502_9 \text{ mol}$$

Now use the total moles (n_{total}) to find the new pressure:

$$P_{final} = \frac{\left(n_{CO} + n_{N_2}\right)RT}{V}$$

$$= \frac{(0.0265_0 \text{ mol} + 0.0502_{9n} \text{ mol}) \left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(245 \text{ K})}{1.15 \text{ L}}$$

$$P_{final} = 1.34 \text{ atm}$$

Into an empty, locked (immoveable) piston with a volume of 1.15 L and temperature of 245 K, you inject:

- 589 mL of carbon monoxide gas at 325 K and 1.20 atm
- 473 mL of nitrogen gas at 298 K and 2.60 atm

If the piston is now allowed to move, what is the new volume after injection?
(Hint: Think about what the final pressure has to be.)

Into an empty, locked (immoveable) piston with a volume of 1.15 L and temperature of 245 K, you inject:
- 589 mL of carbon monoxide gas at 325 K and 1.20 atm
- 473 mL of nitrogen gas at 298 K and 2.60 atm

If the piston is now allowed to move, what is the new volume after injection?
(Hint: Think about what the final pressure has to be.)

First, recognize that the piston's volume will expand to obtain a pressure = 1.00 atm.
Second, realize that temperature (T) and the number of moles (n) are constant.
If so, accept that pressure (P) and volume (V) change.

What we know: $P_1 = 1.34_3$ atm $P_2 = 1.00$ atm
                   $V_1 = 1.15$ L $V_2 = ?$

Set up the expression for these conditions and solve for V_2:

$$P_1 V_1 \ = \ P_2 V_2$$

$$V_2 \ = \ \frac{P_1 V_1}{P_2}$$

$$= \ \frac{(1.34_3 \text{ atm})(1.15 \text{ L})}{(1.00 \text{ atm})}$$

$$V_2 \ = \ 1.54 \text{ L}$$

An experiments shows that 0.495 g sample of an unknown gas occupies 127 mL at 98 °C and 0.992 atm.
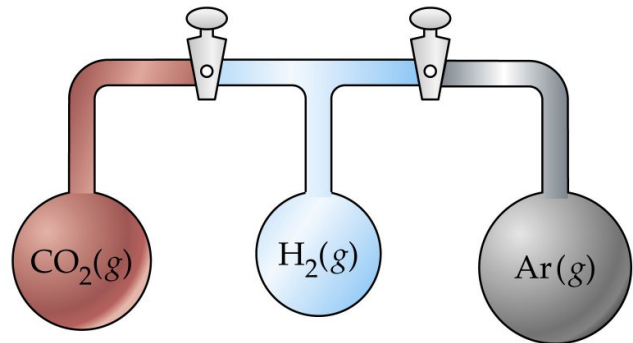*What is the molar mass of the gas?*

Using the ideal gas law, we can solve for the number of moles (n):

$$n = \frac{PV}{RT}$$

$$= \frac{(0.992 \text{ atm})\left(127 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}}\right)}{\left(0.08206 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(371._{15} \text{ K})}$$

$$n = 0.00413_7 \text{ mol}$$

Now we can back-calculate the molar mass (MM) using the mass of the sample:

$$0.495 \text{ g} = 0.00413_7 \text{ mol} \times \frac{MM \text{ g}}{1 \text{ mol}}$$
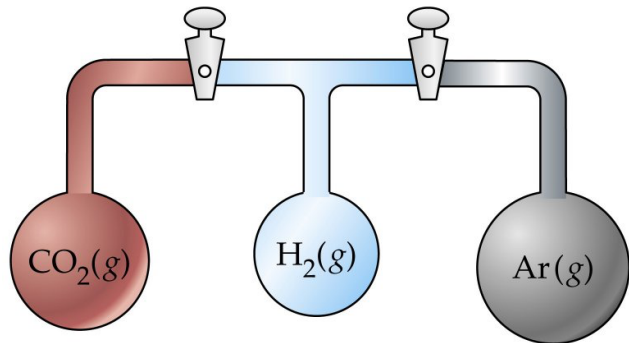
$$MM = 120._{} \frac{\text{g}}{\text{mol}}$$

CO$_2$(g)

H$_2$(g)

Ar(g)

P = 2.13 atm
V = 1.50 L

P = 0.861 atm
V = 1.00 L

P = 1.15 atm
V = 2.00 L

Imagine that both stopcocks were opened so that the gases mix at 298 K.

What is the partial pressure of each gas after opening?

$CO_2(g)$ $H_2(g)$ $Ar(g)$

$P = 2.13$ atm $P = 0.861$ atm $P = 1.15$ atm
$V = 1.50$ L $V = 1.00$ L $V = 2.00$ L

Imagine that both stopcocks were opened so that the gases mix at 298 K.

What is the partial pressure of each gas after opening?

Solve for the number of moles using the pressures of each gas using the ideal gas law:

$$n_{CO_2} = \frac{PV}{RT}$$
$$= \frac{(2.13\ \text{atm})(1.50\ \textbf{L})}{\left(0.08206\ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(298\ \text{K})}$$
$$n_{CO_2} = 0.130_7\ \text{mol}$$

$$n_{H_2} = \frac{PV}{RT}$$
$$= \frac{(0.861\ \text{atm})(1.00\ \text{L})}{\left(0.08206\ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(298\ \text{K})}$$
$$n_{N_2} = 0.0352_1\ \text{mol}$$

$$n_{Ar} = \frac{PV}{RT}$$
$$= \frac{(1.15\ \text{atm})(2.00\ \text{L})}{\left(0.08206\ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(298\ \text{K})}$$
$$n_{Ar} = 0.0940_5\ \text{mol}$$

If we open the stopcocks, the total volume changes to 4.50 L. Now solve for new pressures:

$$P_{CO_2} = \frac{nRT}{V}$$
$$= \frac{(0.130_7\ \text{mol})\left(0.08206\ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(298\ \text{K})}{4.50\ \text{L}}$$
$$P_{CO_2} = 0.710\ \text{atm}$$

$$P_{H_2} = \frac{nRT}{V}$$
$$= \frac{(0.0352_1\ \text{mol})\left(0.08206\ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(298\ \text{K})}{4.50\ \text{L}}$$
$$P_{H_2} = 0.191\ \text{atm}$$

$$P_{Ar} = \frac{nRT}{V}$$
$$= \frac{(0.0940_5\ \text{mol})\left(0.08206\ \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}\right)(298\ \text{K})}{4.50\ \text{L}}$$
$$P_{Ar} = 0.511\ \text{atm}$$

ROOT-MEAN-SQUARE (RMS) SPEEDS

$$u_{\text{rms}} = \sqrt{\frac{3RT}{MM}} \qquad ; \qquad \frac{u_{\text{rms,A}}}{u_{\text{rms,B}}} = \sqrt{\frac{MM_{\text{B}}}{MM_{\text{A}}}} \qquad ; \qquad R = 8.314 \, \frac{\text{kg} \cdot \text{m}^2}{\text{mol} \cdot \text{K} \cdot \text{s}^2}$$

At a given temperature, gases with larger molar masses (MM) move more slowly.

EFFUSION RATE (r_{eff})

$$\frac{r_{\text{eff,A}}}{r_{\text{eff,B}}} = \frac{u_{\text{rms,A}}}{u_{\text{rms,B}}} = \sqrt{\frac{MM_{\text{B}}}{MM_{\text{A}}}}$$

At a given temperature, gases with smaller MMs have a higher chance of escaping.

An unknown gas effuses at a rate of 0.10 m/s and the SO_3 effuses at 0.052 m/s under identical conditions.

What is the molar mass of the unknown gas?

An unknown gas effuses at a rate of 0.10 m/s and the SO_3 effuses at 0.052 m/s under identical conditions.
What is the molar mass of the unknown gas?

Use Graham's Law to find molar mass of gas X:

$$\frac{r_{eff,SO_3}}{r_{eff,X}} = \sqrt{\frac{MM_{SO_3}}{MM_X}}$$

$$\left(\frac{r_{eff,SO_3}}{r_{eff,X}}\right)^2 = \frac{MM_{SO_3}}{MM_X}$$

$$MM_X = MM_{SO_3}\left(\frac{r_{eff,SO_3}}{r_{eff,X}}\right)^2$$

$$MM_X = \left(80.06\ \frac{g}{mol}\right)\left(\frac{0.10\ \frac{m}{s}}{0.052\ \frac{m}{s}}\right)^2$$

$$MM_X = 22\ \frac{g}{mol}$$